



Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at <http://about.jstor.org/participate-jstor/individuals/early-journal-content>.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

Completing the square, using the positive sign of the radical,

$$r(a^2 - b^2)(b^2 \cos^2 \theta - a^2 \sin^2 \theta) = ab(a^2 + b^2)\sqrt{(b^2 \cos^2 \theta + a^2 \sin^2 \theta)} \dots\dots\dots (6).$$

Multiplying both sides of (6) by $r(b^2 \cos^2 \theta - a^2 \sin^2 \theta)$, squaring, and putting in the values from (1) and (2),

$$(a^2 - b^2)^2 (b^2 x^2 - a^2 y^2)^2 - a^2 b^2 (a^2 + b^2)^2 (b^2 x^2 + a^2 y^2) = 0 \dots\dots\dots (7).$$

This is one factor of the given expression. Using the negative sign after completing the square in (3), and employing (4) and (5),

$$(a^2 - b^2)(b^2 \cos^2 \theta - a^2 \sin^2 \theta)r\sqrt{(b^2 \cos^2 \theta + a^2 \sin^2 \theta)}[r\sqrt{(b^2 \cos^2 \theta + a^2 \sin^2 \theta)} + ab] \\ = 0 \dots\dots\dots (8).$$

Equating the last factor to zero, rationalizing, using (1) and (2), we have $a^2 y^2 + b^2 x^2 - a^2 b^2$ as a second factor.

Also solved by G. B. M. Zerr.

251. Proposed by S. A. COREY, Hiteman, Iowa.

$$\text{Prove that } \frac{1}{n+1} + \frac{1}{2(n+2)} + \frac{1}{3(n+3)} + \text{etc.}, = \\ \frac{1}{n^2} + \frac{1}{2} \left[\frac{1}{n-1} + \frac{1}{2(n-2)} + \frac{1}{3(n-3)} + \dots + \frac{1}{l(n-l)} \right],$$

l being equal to $n-1$, n being any positive integer greater than one.

Solution by L. E. NEWCOMB, Los Gatos, Cal.

$$\text{The general term is, } \frac{1}{r(n+r)} = \frac{1}{nr} - \frac{1}{nr(r+n)}. \quad \text{Let } r=1, 2, 3, \dots\dots\dots \text{ in} \\ \text{succession; then } \frac{1}{n+1} = \frac{1}{n} - \frac{1}{n(n+1)}, \quad \frac{1}{2(n+2)} = \frac{1}{2n} - \frac{1}{n(n+2)}, \quad \frac{1}{3(n+3)} \\ = \frac{1}{3n} - \frac{1}{n(n+3)}.$$

\therefore Sum $= \frac{1}{n} - \frac{1}{n(n+1)} + \frac{1}{2n} - \frac{1}{n(n+2)} + \frac{1}{3n} - \frac{1}{n(n+3)} \dots\dots\dots$ and all the terms after the r th vanish.

$$\therefore \text{Sum} = \frac{1}{n} + \frac{1}{2n} + \frac{1}{3n} + \dots + \frac{1}{n^2} \equiv \frac{1}{n^2} + \left[\frac{1}{n} + \frac{1}{2n} + \frac{1}{3n} + \dots + \frac{1}{ln} \right] (1).$$

In the series (2) $\frac{1}{n-1} + \frac{1}{2(n-2)} + \frac{1}{3(n-3)} + \dots + \frac{1}{l(n-l)}$, the general

term is $\frac{1}{r(n-r)} = \frac{1}{nr} + \frac{1}{n(n-r)}$, and since $\frac{1}{n-1} = \frac{1}{n} + \frac{1}{n(n-1)}$, $\frac{1}{2(n-2)} = \frac{1}{2n} + \frac{1}{n(n-2)}$, $\frac{1}{l(n-l)} = \frac{1}{ln} + \frac{1}{n(n-l)} \equiv \frac{1}{n(n-1)} + \frac{1}{n}$, the sum of (2) = $\frac{1}{n} + \frac{1}{2n} + \frac{1}{3n} + \dots + \frac{1}{n} + [\frac{1}{n(n-1)} + \frac{1}{n(n-2)} + \dots + \frac{1}{n}]$. The terms within and without the parenthesis are now plainly identical; consequently, $\frac{1}{2} [\frac{1}{n-1} + \frac{1}{2(n-2)} + \frac{1}{3(n-3)} + \dots + \frac{1}{l(n-l)}]$ substituted for $\frac{1}{n} + \frac{1}{2n} + \frac{1}{3n} + \dots + \frac{1}{ln}$ in the right hand member of (1) will satisfy the equation.

II. Solution by R. D. CARMICHAEL, Hartselle, Alabama.

Represent the series of the first member by S_n . Then,

$$S_2 = \frac{1}{1.3} + \frac{1}{2.4} + \frac{1}{3.5} + \frac{1}{4.6} + \dots = \frac{1}{2} [(1 - \frac{1}{3}) + (\frac{1}{2} - \frac{1}{4}) + (\frac{1}{3} - \frac{1}{5}) + (\frac{1}{4} - \frac{1}{6}) + \dots] = \frac{1}{2} (1 + \frac{1}{2}).$$

$$S_3 = \frac{1}{1.4} + \frac{1}{2.5} + \frac{1}{3.6} + \dots = \frac{1}{3} [(1 - \frac{1}{4}) + (\frac{1}{2} - \frac{1}{5}) + (\frac{1}{3} - \frac{1}{6}) + (\frac{1}{4} - \frac{1}{7}) + \dots] = \frac{1}{3} (1 + \frac{1}{2} + \frac{1}{3}).$$

$$S_n = \frac{1}{n} (1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots + \frac{1}{n}) = \frac{1}{n^2} + \frac{1}{n} (1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots + \frac{1}{n-1})$$

$$= \frac{1}{n^2} + \frac{1}{n} (\frac{1}{n-1} + \frac{1}{n-2} + \frac{1}{n-3} + \dots + \frac{1}{3} + \frac{1}{2} + 1)$$

$$\therefore 2S_n = \frac{2}{n^2} + \frac{1}{n} [(1 + \frac{1}{n-1}) + (\frac{1}{2} + \frac{1}{n-2}) + (\frac{1}{3} + \frac{1}{n-3}) + \dots + (1 + \frac{1}{n-1})].$$

$$\therefore S_n = \frac{1}{n^2} + \frac{1}{2} [\frac{1}{n-1} + \frac{1}{2(n-2)} + \frac{1}{3(n-3)} + \dots + \frac{1}{l(n-l)}], l \text{ being equal to } n-1.$$

Also solved by G. W. Greenwood, Henry Heaton, and G. B. M. Zerr.

252. Proposed by L. E. NEWCOMB, Los Gatos, Cal.

Solve (1) $x - y = \frac{1}{3}\pi$; (2) $\sin x = \cos^3 y$.

Solution by J. SCHEFFER, Hagerstown, Md.

Since $x = y + \frac{1}{3}\pi$, we have $\sin x = \frac{1}{2} \sin y + \frac{1}{2} \sqrt{3} \cos y$.

$$\therefore \frac{1}{2} \sin y + \frac{1}{2} \sqrt{3} \cos y = \cos^3 y.$$

$\therefore 1 - \cos^2 y = 4 \cos^6 y - 4 \sqrt{3} \cos^4 y + 3 \cos^2 y$, or $4 \cos^6 y - 4 \sqrt{3} \cos^4 y + 4 \cos^2 y - 1 = 0$; putting $\cos^2 y = z$, we get $z^3 - \sqrt{3} z^2 + z - \frac{1}{4} = 0$; putting $z = t + \frac{1}{3} \sqrt{3}$, we get $t^3 = \frac{1}{4} - \frac{1}{9} \sqrt{3}$; $\therefore t = (\frac{1}{4} - \frac{1}{9} \sqrt{3})^{\frac{1}{3}}$.